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STRUCTURAL RELIABILITY THEORY, PAPER NO. 1

J. D. SØRENSEN & P. THOFT-CHRISTENSEN
MODEL UNCERTAINTY FOR BILINEAR HYSTERETIC SYSTEMS
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Intergenerational Transmission of Child Sexual Abuse: A Review of the Literature

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Abstract This review examines the literature on the intergenerational transmission of child sexual abuse (CSA). The review is organized into three main sections: (a) the impact of CSA on victims, (b) the impact of CSA on perpetrators, and (c) the impact of CSA on the next generation.

Keywords child sexual abuse, intergenerational transmission, victimization, perpetration, next generation

Child sexual abuse (CSA) is a global public health problem that affects millions of children each year (Finkelhor & Browne, 1985; Finkelhor & Browne, 1985; Finkelhor & Browne, 1985).

The impact of CSA on victims is well documented, with victims experiencing a range of psychological and physical problems (Finkelhor & Browne, 1985; Finkelhor & Browne, 1985; Finkelhor & Browne, 1985).

The impact of CSA on perpetrators is also well documented, with perpetrators experiencing a range of psychological and physical problems (Finkelhor & Browne, 1985; Finkelhor & Browne, 1985; Finkelhor & Browne, 1985).

The impact of CSA on the next generation is also well documented, with victims of CSA being at an increased risk of becoming perpetrators of CSA (Finkelhor & Browne, 1985; Finkelhor & Browne, 1985; Finkelhor & Browne, 1985).

This review examines the literature on the intergenerational transmission of CSA. The review is organized into three main sections: (a) the impact of CSA on victims, (b) the impact of CSA on perpetrators, and (c) the impact of CSA on the next generation.

Child Sexual Abuse: A Global Public Health Problem

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1. INTRODUCTION

In structural reliability analysis at least three types of uncertainty must be considered, namely physical uncertainty, statistical uncertainty, and model uncertainty (see e.g. Thoft-Christensen & Baker [1]). The physical uncertainty is usually modelled by a number of basic variables. The statistical uncertainty - due to lack of information - can e.g. be taken into account by describing the variables by predictive density functions, Veneziano [2].

In general, model uncertainty is the uncertainty connected with mathematical modelling of the physical reality.

When structural reliability analysis is related to the concept of a failure surface (or limit state surface) in the n -dimensional basic variable space then model uncertainty is at least due to the neglected variables, the modelling of the failure surface and the computational technique used. A more precise definition is given in section 2, where some different methods to treat model uncertainty are described. In section 3 a new method based on subjectively modelled conditional density functions is presented. It is shown that in some special cases this method is equivalent to existing more simple methods.

In the analysis of dynamically loaded structures it is often assumed that the loading and the response can be modelled by stationary stochastic processes. Further, it is assumed that the structures can be modelled by non-linear systems showing hysteresis. This non-linear behaviour is essential to the design procedure from an economic and reliability point of view. In section 4 it is shown how the probability of failure of a simple bilinear oscillator can be estimated and in section 5 it is demonstrated by numerical examples how model uncertainty can be included in the calculations.

2. MODEL UNCERTAINTY

Usually mathematical models describing the relations between the basic variables are deterministic models although there is a great deal of uncertainty associated with them. They may be based on a good understanding of the mechanical problem, but they will usually to some degree be imperical. In this paper model uncertainty is uncertainty in relation to the stochastic structure of the basic variables and the choice of failure surface. The last-mentioned uncertainty in relation to the failure surface is due to a number of neglected variables and also to the mathematical expressions chosen. The choice of density functions for the basic variables is of great importance due to tail sensitivity. Usually very little is known of the shape of the density functions in the significant intervals. Selection of density functions has been treated by Grigoriu, Veneziano & Cornell [11].

It has been suggested to evaluate the model uncertainty by comparing different mathematical models or using experimental data from existing structures or laboratory tests. However, this type of comparison will also be uncertain. Use of experience from other types of structure will also be uncertain due to the great variety of structures of interest.

It is clearly of importance to include model uncertainty in such a way that the estimation of the structural reliability is not getting too complicated. Further model uncertainty should be included in a form which is invariant to mathematical transformations of the equations in question. As emphasized by Ditlevsen [7] this is obtained if the model uncertainty can be related directly to the basic variables.

Model uncertainty can be included in level 1 methods simply by adding constants to the basic variables or multiplying the basic variables by constants. The magnitude of these constants must be determined by a subjective judgement of the model uncertainty. In level 2 methods (first order - second moment methods) the same technique may be used if the constants are substituted by stochastic variables with subjectively modelled second order moments (see Ang [3], Milford [4], NKB-recommendations [5]).

Let the idealized failure surface be given by

$$g(\underline{x}) = 0 \quad (1)$$

where \underline{x} is a realization of the basic variables $\underline{X} = (X_1, \dots, X_n)$. Ditlevsen [7] has suggested that the model uncertainty can be modelled by assuming the failure surface to be stochastic in the basic variable space. According to the extended level 2 methods the basic variables are transformed into variables which are normally distributed with the expected values $\underline{\mu}_X$ and the matrix of covariance \underline{C}_X (see Ditlevsen [8]). In this space the stochastic failure surface is modelled by a linear transformation of the normally distributed variables

$$\underline{M}(\underline{X}) = \underline{A} \underline{X} + \underline{Z} \quad (2)$$

where \underline{A} is a matrix with constant elements and \underline{Z} a normally distributed stochastic vector with expected value $\underline{\mu}_Z$ and covariance $\underline{C}_Z(\underline{x})$. By letting \underline{C}_Z be dependent on \underline{x} local variations in the model uncertainty can be included. The parameters in (2) are determined subjectively. The stochastic failure surface is defined by $g(\underline{M}(\underline{x})) = 0$. Assume \underline{X} and \underline{Z} to be independent, then the expected value and covariance of $\underline{M}(\underline{X})$ are $\underline{A}\underline{\mu}_X + \underline{\mu}_Z$ and $\underline{A}\underline{C}_X\underline{A}^T + \underline{C}_Z$, respectively. Therefore, by this technique the model uncertainty is included with only a small increase in the calculation time. This formulation is in principle equal to the method suggested in the NKB-regulations [5].

Several techniques to include model uncertainty have been suggested in level 3 reliability methods. Consider a number of mathematical models with corresponding reliability estimates. Let the probability that such a model is correct be given by a formal probability. Then a weighted estimate of the reliability can be constructed by the total probability theorem. A second technique is based on modelling the uncertain variables by fuzzy sets (see Blockley & Ellison [9]). The model uncertainty is included in this technique by subjectively modelled conditional fuzzy sets characterized by membership functions.

3. A MODELLING METHOD BASED ON CONDITIONAL DENSITY FUNCTIONS

This section describes a method to include model uncertainty in level 3 reliability methods, where the model uncertainty is modelled by conditional density functions $f_{\underline{X}'|\underline{X}}(\underline{x}'|\underline{x})$, where $\underline{X}' = (X'_1, \dots, X'_n)$ can be considered as stochastic variables modelled in such a way that the physical uncertainty corresponding to \underline{X} and the model uncertainty are included in their characterization. The conditional density function is determined on the basis of subjective estimates and should be chosen in such a way that the calculation of the reliability is only slightly increased. Note that the model uncertainty is modelled in the basic variable space so that this formulation is invariant to transformations of the failure function (1). The failure probability is now given by

$$P_f = \int_{g(\underline{x}) \leq 0} \int_{R^n} f_{\underline{X}'|\underline{X}}(\underline{x}'|\underline{x}) f_{\underline{X}}(\underline{x}) d\underline{x} d\underline{x}' \quad (3)$$

If model uncertainty can be neglected then

$$f_{\underline{X}'|\underline{X}}(\underline{x}'|\underline{x}) = \delta(\underline{x}' - \underline{x}) \quad (4)$$

where $\delta(\cdot)$ is Dirac's delta-function, and the probability of failure is defined by the usual expression

$$P_f = \int_{g(\underline{x}) \leq 0} f_{\underline{X}}(\underline{x}) d\underline{x} \quad (5)$$

Evaluation of the multi-integral in (3) will of course in general be very expensive. However, if the conditional density function is chosen in such a way that $f_{\underline{X}'}$ can be calculated without using numerical integration then inclusion of model uncertainty will not increase the computer time. The probability of failure can then be calculated by a formula like (5). This procedure corresponds to using natural conjugated families of density functions in Bayesian statistics.

Let \underline{X} be normally distributed with expected values $\underline{\mu}_X$ and covariance \underline{C}_X

$$f_{\underline{X}}(\underline{x}) = N(\underline{x}; \underline{\mu}_X, \underline{C}_X) \quad (6)$$

and let the conditional density function be given by

$$f_{\underline{X}'|\underline{X}}(\underline{x}'|\underline{x}) = N(\underline{x}'; \underline{\mu}_X^* + \underline{A}\underline{x}, \underline{C}_X^*) \quad (7)$$

where $\underline{\underline{A}}$ is a quadratic matrix with constant elements. By the assumption (7) the model uncertainty is included through a linear transformation from $\underline{\underline{X}}$ to $\underline{\underline{X}}'$ and is modelled by $\underline{\underline{\mu}}_{\underline{\underline{X}}}^*$, $\underline{\underline{A}}$ and $\underline{\underline{C}}_{\underline{\underline{X}}}^*$. By inserting (6) and (7) in (3) one gets

$$P_f = \int_{g(\underline{\underline{x}}') \leq 0} N(\underline{\underline{x}}'; \underline{\underline{\mu}}_{\underline{\underline{X}}}^* + \underline{\underline{A}}\underline{\underline{\mu}}_{\underline{\underline{X}}} + \underline{\underline{C}}_{\underline{\underline{X}}}^* + \underline{\underline{A}}\underline{\underline{C}}_{\underline{\underline{X}}}\underline{\underline{A}}^T) d\underline{\underline{x}}' \quad (8)$$

This formulation is equivalent to the modelling (2) if $\underline{\underline{\mu}}_{\underline{\underline{X}}}^* = \underline{\underline{\mu}}_{\underline{\underline{Z}}}$ and $\underline{\underline{C}}_{\underline{\underline{X}}}^* = \underline{\underline{C}}_{\underline{\underline{Z}}}$. The failure probability (8) can be calculated either by numerical integration or by e.g. the extended first order second moment method (see [8]). Local variations in the model uncertainty can e.g. be included by letting $\underline{\underline{\mu}}_{\underline{\underline{X}}}^*$ and $\underline{\underline{C}}_{\underline{\underline{X}}}^*$ be dependent on $\underline{\underline{x}}$.

P_f can also be written

$$P_f = \int_{\mathbb{R}^n} f_{\underline{\underline{X}}|\underline{\underline{X}}}(\underline{\underline{x}}|\underline{\underline{x}}) \int_{g(\underline{\underline{x}}') \leq 0} f_{\underline{\underline{X}}'|\underline{\underline{X}}}(\underline{\underline{x}}'|\underline{\underline{x}}) d\underline{\underline{x}}' d\underline{\underline{x}} = \int_{\mathbb{R}^n} f_{\underline{\underline{X}}}(\underline{\underline{x}}) P(\underline{\underline{x}}) d\underline{\underline{x}} \quad (9)$$

where

$$P(\underline{\underline{x}}) = \int_{g(\underline{\underline{x}}') \leq 0} f_{\underline{\underline{X}}'|\underline{\underline{X}}}(\underline{\underline{x}}'|\underline{\underline{x}}) d\underline{\underline{x}}' \quad (10)$$

is the failure probability given $\underline{\underline{x}}$. If the model uncertainty is neglected then

$$P(\underline{\underline{x}}) = \begin{cases} 1 & \text{if } g(\underline{\underline{x}}) \leq 0 \\ 0 & \text{if } g(\underline{\underline{x}}) > 0 \end{cases} \quad (11)$$

Example 1

Let $n = 2$ and X_1 and X_2 be independent and normally distributed $N(0, 1)$. Let $f_{\underline{\underline{X}}'|\underline{\underline{X}}}(\underline{\underline{x}}'|\underline{\underline{x}}) = N(\underline{\underline{x}}'; \underline{\underline{x}}, \sigma) \cdot N(\underline{\underline{x}}_2; \underline{\underline{x}}_2, \sigma)$ and let the failure event be defined by $\{x_1, x_2 | x_1 + 3 \leq 0 \vee x_2 + 3 \leq 0\}$.

Then from (9)

$$P_f = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} N(x_1; 0, 1) N(x_2; 0, 1) P(x_1, x_2) dx_1 dx_2 \quad (12)$$

where

$$P(x_1, x_2) = 1 - (1 - \Phi(\frac{-3 - x_1}{\sigma}))(1 - \Phi(\frac{-3 - x_2}{\sigma})) \quad (13)$$

The function P is shown in figure 1 for $\sigma = 0, 0.5$, and 1 . For $\sigma = 0$ (corresponding to no model uncertainty) $P(x_1, x_2)$ is a step function. For increasing σ the function $P(x_1, x_2)$ is becoming still »smoother». Therefore, alternatively, the model uncertainty can be modelled by prescribing $P(\underline{\underline{x}})$. By this formulation $P(\underline{\underline{x}})$ is a function modelling the failure surface uncertainty. In the same figure the generalized reliability index β_s is shown. As expected β_s is decreasing with increasing σ (increasing model uncertainty).

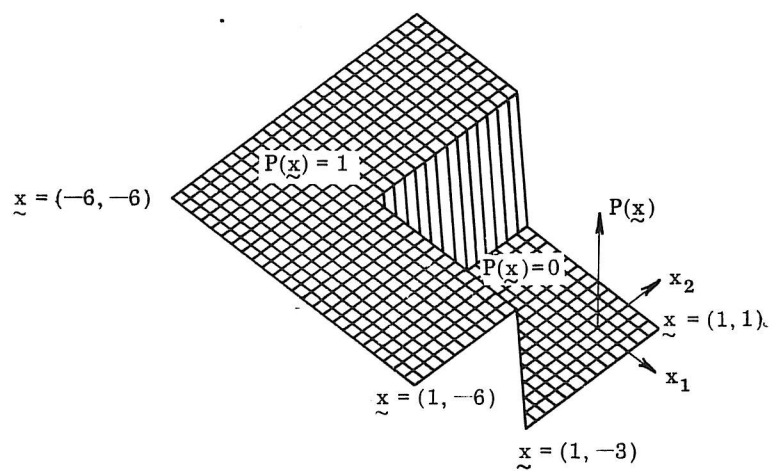
4. ELASTO-PLASTIC HYSTERETIC SYSTEMS

This section describes a method to estimate the reliability of a structure which can be modelled as an elasto-plastic simple oscillator loaded by white-noise. Failure is defined as the event that the permanent deformation crosses a critical deterministic value b . The yield limit of the oscillator is called y .

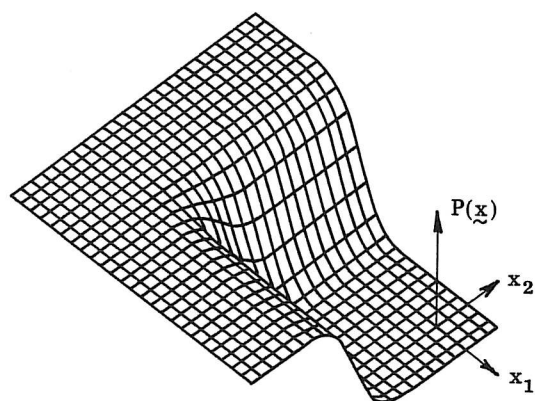
The equation of motion for a linear time invariant system with one degree of freedom is

$$\ddot{x}_0 + 2\zeta_0\omega_0\dot{x}_0 + \omega_0^2x_0 = f \quad (14)$$

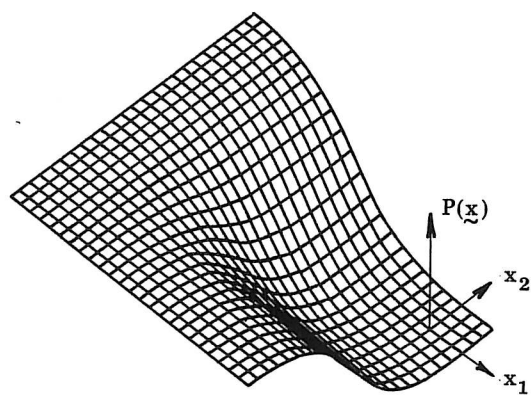
where ω_0 is the undamped eigenfrequency, ζ_0 the damping ratio, $x_0(t)$ a realization of the response process $\{X_0(t), t \in [0, \infty[\}$ and $f(t)$ a realization of the load process $\{F(t), t \in [0, \infty[\}$. The load process is assumed to be stationary and Gaussian with the spectrum $S_F(\omega) = s_F$. Then the stationary variance of X_0 is



a) $\sigma = 0, \beta_s = -\Phi^{-1}(P_f) = 2.78$



b) $\sigma = 0.5, \beta_s = -\Phi^{-1}(P_f) = 2.44$



c) $\sigma = 1, \beta_s = -\Phi^{-1}(P_f) = 1.83$

Figure 1. The function $P(x_1, x_2)$.

$$\sigma_{X_0}^2 = \frac{\pi S_F}{2\zeta_0 \omega_0^3} \quad (15)$$

For slightly damped structures realizations of the response process show that peaks outside $]-y, y[$ tend to come in «clumps». Vanmarche [12] has suggested the following approximation for the expected number $E[N_y]$ of consecutive peaks outside $]-y, y[$

$$E[N_y] = [1 - \exp(-\sqrt{\frac{\pi}{2}} r q)]^{-1} \quad (16)$$

where $r = y/\sigma_X$ and $q \cong 2\sqrt{\zeta_0/\pi}$. The expected number of «clumps» per unit time can be determined by, Vanmarche [12],

$$\mu_y = \frac{\omega_0}{\pi} (1 - \exp(-\sqrt{\frac{\pi}{2}} r q)) \exp(-\frac{r^2}{2}) \quad (17)$$

Next it is assumed that important statistical characteristics for an elasto-plastic system can be determined from a linear system with the equivalent system characteristics ζ_e and ω_e calculated by the method of Krylov and Boguliubov (see Caughey [13]). It is here assumed that the response is narrow-banded and the expected accumulated permanent deformation is 0. By using the Stratonovitch-Khasminskii limit theorem Roberts [14] has shown on the same assumptions that the simultaneous density function for the amplitude A and the phase Ψ for the narrow-banded response process of the elasto-plastic system is

$$f_\Psi(\psi)f_A(a) = \begin{cases} \frac{1}{2\pi} ca \exp(-\frac{a^2}{2\sigma_0^2}) & , 0 \leq a \leq y, \psi \in [0, 2\pi] \\ \frac{1}{2\pi} ca \exp(-\frac{a^2}{2\sigma_0^2} - \frac{a-y}{\delta})(\frac{a}{y})^{y/\delta} & , a > y, \psi \in [0, 2\pi] \end{cases} \quad (18)$$

where

$$\sigma_0^2 = \sigma_{X_0}^2 (\frac{\omega_0}{\omega_e})^2 \quad (19)$$

$$\delta = \frac{\pi \zeta_0 \omega_0 \sigma_{X_0}^2}{2\omega_e y} \quad (20)$$

By using (18) the distribution function F_V for the velocity of the response by crossing into the plastic area can be determined (see Stratonovich [15])

$$F_V(v) = \frac{F_A(\sqrt{y^2 + (v/\omega_e)^2}) - F_A(y)}{1 - F_A(y)} \quad (21)$$

Let the response cross into the plastic area at the time $t = t_0$ with the velocity V and leave it again at $t = t_0 + \Delta T$. If the damping and inertia terms in the energy equation are neglected then it can be proved that

$$F_{\Delta D_0}(d) \cong F_V(\omega_0 \sqrt{2yd}) \quad , d > 0 \quad (22)$$

where $F_{\Delta D_0}$ is the distribution function for the increment in plastic deformation ΔD_0 . It is seen from (19) that f_A for $y/\sigma_{X_0} \rightarrow \infty$ is Rayleigh distributed. (22) then shows that ΔD_0 is exponentially distributed with the expected value $\sigma_{X_0}^2/y$.

Let ΔD be the permanent deformation from a single outcrossing of the elastic range. It is then reasonable to approximate $f_{\Delta D}$ by the Laplace distribution

$$f_{\Delta D}(d) = \frac{1}{\sqrt{2} \sigma_{\Delta D}} \exp(-\frac{|d| \sqrt{2}}{\sigma_{\Delta D}}) \quad (23)$$

where

$$\sigma_{\Delta D}^2 = \int_0^\infty x^2 f_{\Delta D_0}(x) dx \quad (24)$$

The accumulated permanent deformation $D(t)$, $t \in]0, T]$ can be written

$$D(T) = \sum_{i=1}^{N(T)} \Delta D_i^* \quad (25)$$

where $\{N(t), t \in]0, T]\}$ is a stochastic counting process, and ΔD_i^* is the increment from «clump» number i . When y is large compared with σ_{X_0} then it is reasonable to expect this counting process to be a Poisson process with the intensity ν and to assume the individual terms in (25) to be independent. $D(t)$ is then modelled by a filtered Poisson process.

Let $f_{\Delta D^*}$ be given by (23). The characteristic function for ΔD^* is then

$$\varphi_{\Delta D^*} = (1 + \frac{1}{2} (\sigma_{\Delta D^*} u)^2)^{-1} \quad (26)$$

and the density function $f_{D_n} = f_{D(T)}$ for $N(T) = n$ is

$$\begin{aligned} f_{D_n}(x) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-iux} (1 + \frac{1}{2} (\sigma_{\Delta D^*} u)^2)^{-n} du \\ &= \frac{1}{\pi} \int_0^{\infty} (1 + \frac{1}{2} (\sigma_{\Delta D^*} u)^2)^{-n} \cos ux \, du = \frac{\sqrt{2}}{\sigma_{\Delta D^*} \sqrt{\pi}} \left(\frac{x}{\sqrt{2} \sigma_{\Delta D^*}} \right)^{n-\frac{1}{2}} K_{n-\frac{1}{2}} \left(\frac{\sqrt{2} x}{\sigma_{\Delta D^*}} \right) \end{aligned} \quad (27)$$

where K_n is the modified Bessel function. For $\nu \rightarrow \infty$, $\sigma_{\Delta D^*} \rightarrow 0$ and $\sigma_{\Delta D^*}^2 \nu$ constant it is seen that

$$\log \varphi_{D(T)}(u) \rightarrow -\frac{1}{2} \sigma_{\Delta D^*}^2 \nu T \quad (28)$$

(28) shows that $D(T)$ is normally distributed $N(\cdot; 0, \sigma_{\Delta D^*} \sqrt{\nu T})$ at time T . If it is assumed that $D(0) = 0$ and that $D(t)$ has stationary independent increments, then $\{D(t)\}$ is approximating a Wiener process when the number of outcrossings approaches ∞ and the standard deviation for the increments approaches 0.

The intensity ν is put equal to μ_y (see (17)) and the standard deviation $\sigma_{\Delta D^*}$ of the increment in permanent deformation per «clump» is chosen approximately equal to $\sqrt{E[N_y]} \sigma_{\Delta D}$, where $E[N_y]$ is given by (16).

The elasto-plastic oscillator is assumed to fail if the accumulated permanent deformation leaves the safe interval $S =]-b, b[$. It is shown by Nielsen, Sørensen and Thoft-Christensen [16] that the first passage density $f_0(t)$ can be determined on the basis of the Markov property of the process

$$f_0(t) = \nu \sum_{n=0}^{\infty} \frac{(\nu t)^n \exp(-\nu t)}{n!} (1 - \lambda_n) \prod_{\ell=0}^{n-1} \lambda_{\ell} \quad (29)$$

where

$$\lambda_{\ell} = - \frac{\int_{-b}^b f_{D_n}(x) \int_{-b}^b f_{\Delta D^*}(x-z) dz dx}{\int_{-b}^b f_{D_n}(x) dx} \quad (30)$$

The probability of failure in the time interval $]0, T[$ can be estimated by

$$P_f(t) = \int_0^T f_0(t) dt \quad (31)$$

if $D_0 \in S$.

Example 2

Let $\xi_0 = 0.02$, $\omega_0 = 10 \pi s^{-1}$, $b = 0.02$, $\sigma_{X_0} = 0.04$, $y/\sigma_{X_0} = 2.5$, and $P(D(0) = 0) = 1$. It is seen from (17) and (24) that $\nu = 0.173$ and $\sigma_{\Delta D^*} = 0.0105$. The estimate (31) is shown in figure 2 and compared with simulation estimates. The simulation estimates are shown as vertical bars corresponding to the 95% confidence intervals. The small horizontal bars are the point estimates. In this example the agreement is very good.

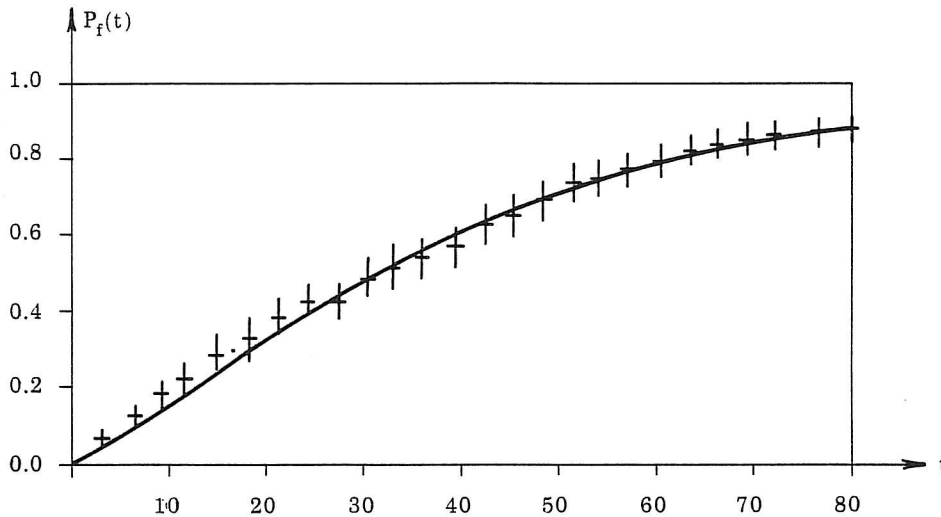


Figure 2. Failure probability $P_f(t)$ for an elasto-plastic hysteretic system.

5. APPLICATIONS OF MODELLING OF MODEL UNCERTAINTY

This section shows in two examples how the modelling method introduced in section 3 can be used for elasto-plastic hysteretic systems.

Example 3

The same system as in example 2 is considered but now model uncertainty is taken into account. Model uncertainty can in this case e.g. be due to

- the stress-strain relation is not perfectly elasto-plastic
- the response cannot be modelled sufficiently accurately by the first mode of vibration, i.e. the structure should be modelled by an n-dimensional system.

Let the permanent deformation ΔD^* at a single «clump» be a basic variable with a density function given by (23). The conditional density function in (3) is chosen as

$$f_{\Delta D'|\Delta D^*}(d'|d) = \frac{1}{2s} \exp\left(-\frac{1}{s} |d' - d|\right) \quad (32)$$

where the influence from the model uncertainty is eliminated for $s \rightarrow 0$. From (23) and (32) it is seen that

$$f_{\Delta D'}(d) = \int_{-\infty}^{\infty} f_{\Delta D'|\Delta D^*}(d|x) f_{\Delta D^*}(x) dx$$

$$= \begin{cases} \frac{\frac{1}{2s} \exp(-\frac{|d|}{s})}{1 - (\frac{\sigma_{\Delta D^*}}{\sqrt{2}s})^2} + \frac{\frac{1}{\sqrt{2}\sigma_{\Delta D^*}} \exp(-\frac{\sqrt{2}|d|}{\sigma_{\Delta D^*}})}{1 - (\frac{\sqrt{2}s}{\sigma_{\Delta D^*}})^2} & \text{for } s \neq \frac{\sigma_{\Delta D^*}}{\sqrt{2}} \\ \frac{1}{4s} (1 + \frac{|d|}{s}) \exp(-\frac{|d|}{s}) & \text{for } s = \frac{\sigma_{\Delta D^*}}{\sqrt{2}} \end{cases} \quad (33)$$

Note that in accordance with (3) and (5) one gets for $s \rightarrow 0$ that

$$f_{\Delta D'}(d) \rightarrow \frac{1}{\sqrt{2}\sigma_{\Delta D^*}} \exp\left(-\frac{\sqrt{2}|d|}{\sigma_{\Delta D^*}}\right) \quad (34)$$

The conditional density function $f_{\Delta D'|\Delta D^*}$ and the density function $f_{\Delta D'}$ are shown in figure 3 for different values of s ($\sigma_{\Delta D^*} = 0.0105$). The probability of failure $P_f(t)$ is shown in figure 4 for $s = 0, 0.005$ and 0.01 . It is seen from figure 4 that the model uncertainty has most influence with small failure probabilities. The computer calculation is increased very little in this example, since (33) can be used directly instead of $f_{\Delta D^*}$ in (27) - (31).

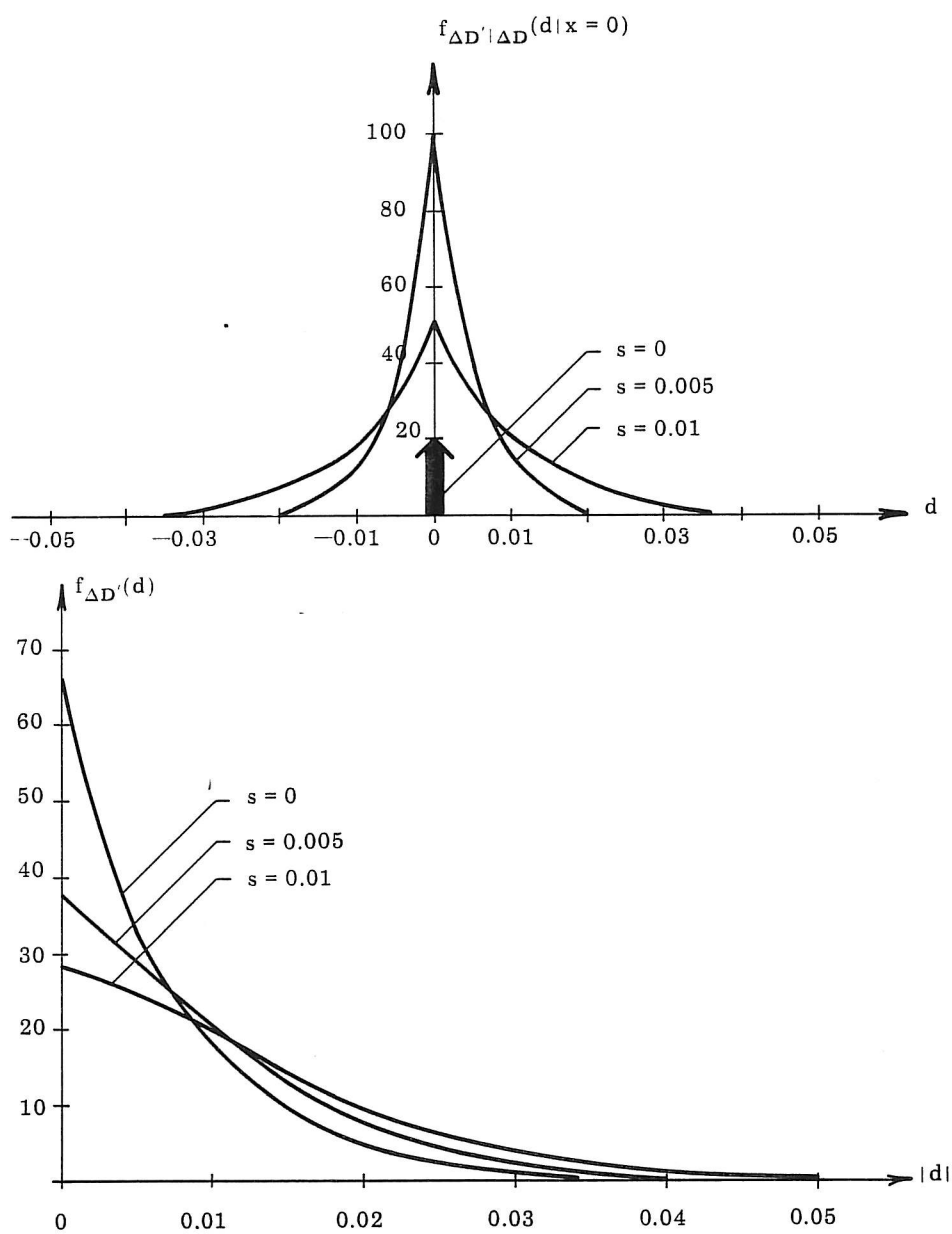


Figure 3. Density functions $f_{\Delta D'|\Delta D}$ and $f_{\Delta D'}$.

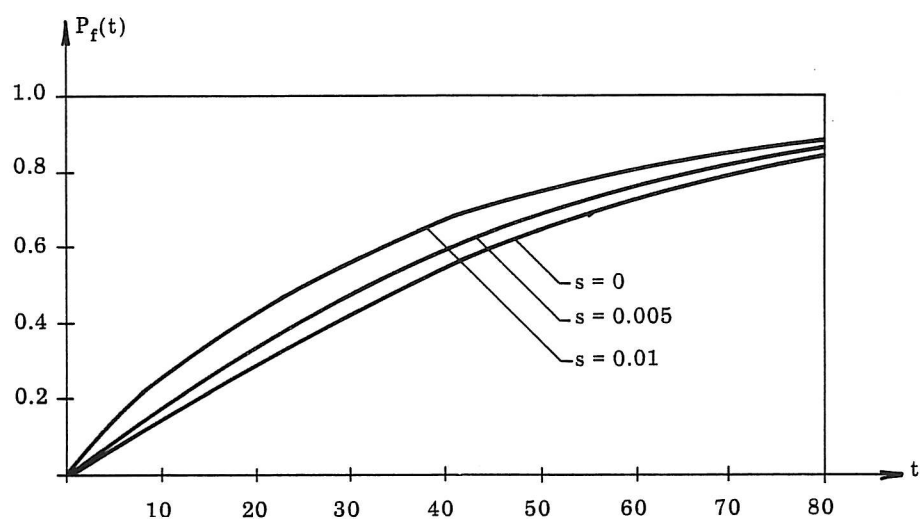


Figure 4. The failure probability for different levels of model uncertainty.

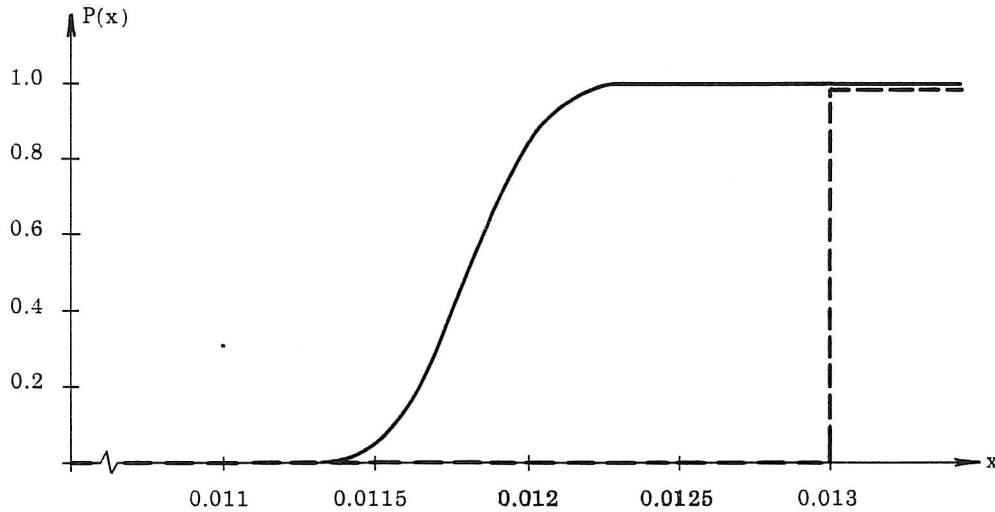


Figure 5. ——— with model uncertainty included
 - - - without model uncertainty included.

Example 4

Consider the case where the probability of getting more than one upcrossing in the time interval $]0, T[$ is negligible. Let the failure function be

$$g(d) = d - d^* \quad , \quad d \geq 0 \quad (35)$$

where d^* is the maximum permissible permanent deformation by a single outcrossing of the elastic region and where d is a realization of the stochastic variable ΔD_0 (see (22)). Let the model uncertainty be modelled by the conditional density function

$$f_{\Delta D'_0 | \Delta D_0}(d'_0 | d_0) = N(d'_0; 1.1 d_0, 0.0002) / \Phi(5500 d_0) \quad , \quad d' > 0, d > 0 \quad (36)$$

Then it is seen from (9) that

$$P_f = \int_{d^*}^{\infty} \int_0^{\infty} f_{\Delta D'_0 | \Delta D_0}(x' | x) f_{\Delta D}(x) dx dx' = \int_0^{\infty} f_{\Delta D_0}(x) P(x) dx \quad (37)$$

where

$$P(x) = 1 - \frac{\Phi\left(\frac{d^* - 1.1 x}{0.0002}\right)}{\Phi(5500 x)} \quad (38)$$

$P(x)$ is shown in figure 5 for $d^* = 0.013$. $P(x)$ corresponding to negligible model uncertainty is shown as a step function. The failure probability $P_f = 0.202$ when model uncertainty is included and $P_f = 0.162$ without model uncertainty included.

6. CONCLUSIONS

A new model to include model uncertainty in reliability analysis is presented and investigated in some details. The model uncertainty is modelled by a conditional density function. If this density function is chosen in an appropriate way corresponding to naturally conjugated density functions in Bayesian statistics then the computational work is only slightly increased. The model uncertainty is modelled in the physical basic space so that invariance is secured.

Further a method to estimate the reliability of structures modelled by a simple elasto-plastic oscillator is presented. The accumulated permanent deformations are modelled by a Markov process and failure defined by crossing of a critical limit. This method is compared with simulation estimates and good agreement is achieved for a structure with slight damping.

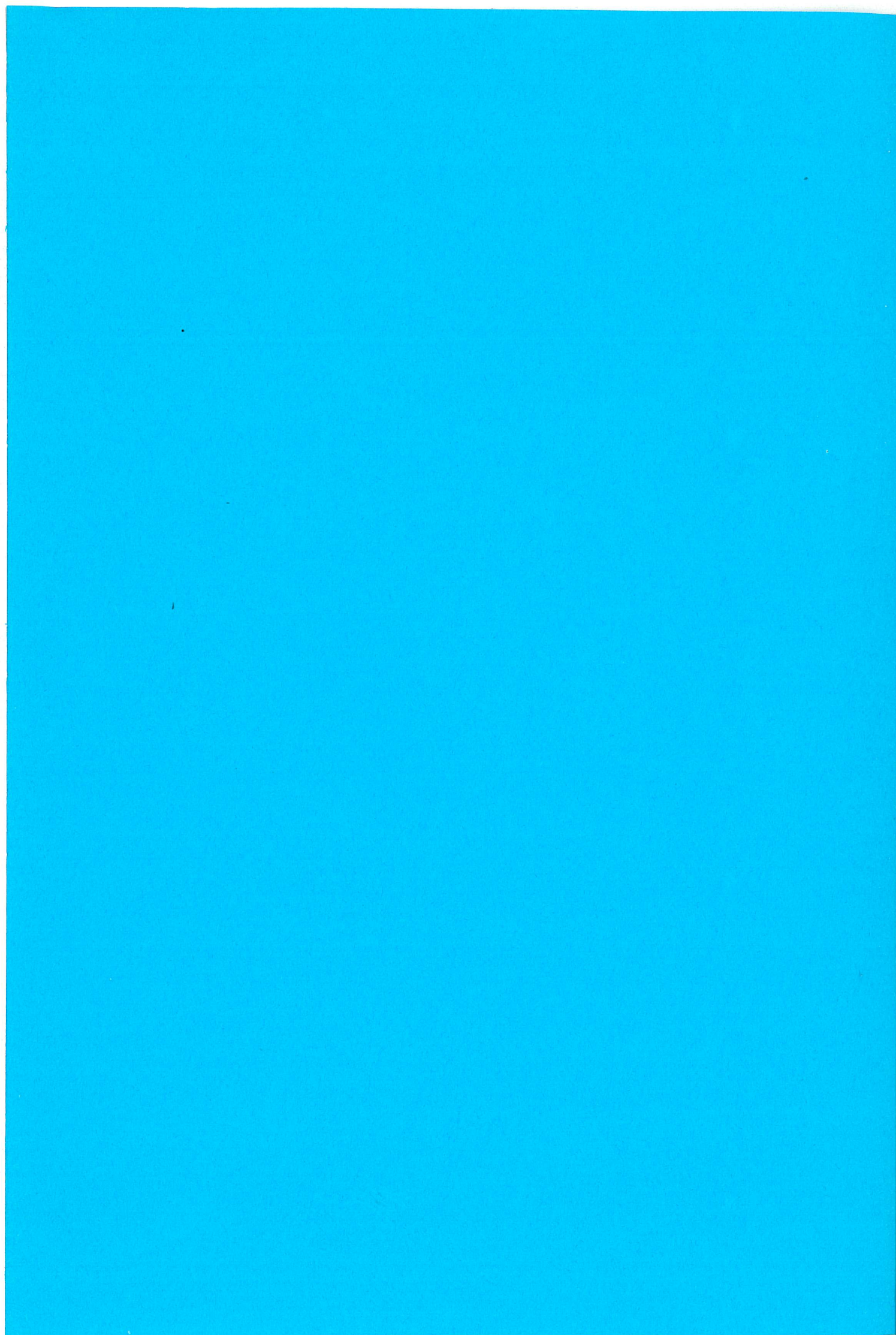
Finally, it is demonstrated by two examples how model uncertainty can be included in the estimate of reliability of elasto-plastic oscillators.

7. ACKNOWLEDGEMENT

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